Exercise 15

Calculate y'.

$$y + x\cos y = x^2 y$$

Solution

Take the derivative of both sides with respect to x.

$$\frac{d}{dx}(y + x\cos y) = \frac{d}{dx}(x^2 y)$$

$$\frac{d}{dx}(y) + \frac{d}{dx}(x\cos y) = \frac{d}{dx}(x^2 y)$$

Apply the product rule.

$$\frac{dy}{dx} + \left[\frac{d}{dx}(x)\right]\cos y + x\left[\frac{d}{dx}(\cos y)\right] = \left[\frac{d}{dx}(x^2)\right] y + x^2\left[\frac{d}{dx}(y)\right]$$

$$\frac{dy}{dx} + (1)\cos y + x\left[(-\sin y)\cdot\frac{d}{dx}(y)\right] = (2x)y + x^2\left(\frac{dy}{dx}\right)$$

$$\frac{dy}{dx} + \cos y + x\left(-\sin y\cdot\frac{dy}{dx}\right) = 2xy + x^2\frac{dy}{dx}$$

Bring all terms with dy/dx to one side.

$$\cos y - 2xy = x^2\frac{dy}{dx} - \frac{dy}{dx} + x\sin y\frac{dy}{dx}$$

$$\cos y - 2xy = (x^2 - 1 + x\sin y)\frac{dy}{dx}$$

Therefore,

$$\frac{dy}{dx} = \frac{\cos y - 2xy}{x^2 - 1 + x\sin y}.$$